# Mathématiques chins <br> Frontières 

## SUMMARY

## You will find in this file:

- the rules of the competition
(modified in September 2019);
- instructions for the conduct of the 2023 discovery event;
- the discovery test 2023 ;
- elements of solutions;
- a proposed marking scheme including the objectives and competencies of the exercises.

For any remarks, suggestions, proposed exercise ideas :
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# Mathématiques Frontières COMPETITION RULES MATHEMATICSWITHOUT FRONTIERS Scotland (and rUK) Classes 

Mathématiques sans Frontières is a competition for National 5 and Higher grade students. The test consists of collectively solving ten exercises (for Nat. 5) and thirteen (for Higher). It is not an individual competition.

Teams must consist of classes formed for the teaching of mathematics of the current year; they cannot be teams set up specifically for the Mathematics without Borders competition.

However, the presence of a small number of pupils from higher-level classes is allowed during the final test, if it does not lead to a significant increase in class size. Under no circumstances can it be an entire class of correspondents. The teacher supervising the test must mention on the submission sheet the presence of additional pupils, specifying their number. These higherlevel pupils who participated in the event will not receive a prize.

Mathématiques sans Frontières is a competition giving rise to a winner, with a prize available for Scottish classes (donated, with thanks, from the Scottish Mathematical Council): every precaution must be taken to avoid leaks and cheating. The final test must take place on a date and in a time slot that were defined the previous year in international assembly. In case of unavailability of the class on the fixed date, the test can be taken after this date but never before.

This year, the final test should take place on Thursday 9th March, 2023.

## Organization of the final test:

- Each participating class composes in an classroom (without access to computers).
- Students may be supervised by any teacher in the school, including their mathematics teacher. All classes in the same institution must work in the same time slot.
- The students organize themselves as the
- $y$ wish: they can talk to each other, circulate in the room made available to them, work in groups, use the blackboard, ... taking care not to interfere with neighbouring classes.
- Each class returns one answer sheet per exercise.
- The solution of the foreign language exercise must be written in one of the languages in which it is stated.
- No student can pick up anything outside the room once the test has begun.


## Authorized Equipment:

- Calculators (*)
- Drawing instruments
- Dictionaries and atlases (dictionary and paper atlas; electronic form excluded)
- Bilingual dictionaries (paper dictionary; electronic form excluded)
- Small stationery equipment and draft sheets
- Class textbooks and student notebooks
$\left(^{*}\right)$ Calculators must be standalone. If they have a means of communicating, it must be disabled.

Unauthorized Material:

- Phones, tablets and any device to communicate.
- Translators.
- Computers.

The organizing teams reserve the right to disqualify any class that has not respected the rules of the competition.

MATHÉMATIQUES SANS FRONTIÈRES INSTRUCTIONS FOR THE DISCOVERY TEST

## Preamble

This event does not count towards the final ranking; It must allow the class to be trained for the final competition on Thursday, March 9, 2023.

For this training to be formative, it is desirable that the mathematics teacher supervises his class at least during the 1st hour and that they assist the students in the organization of their research. He can help to remove blockages and allow them to succeed.

## Course of the test

Students organize themselves as they wish to work; They can talk to each other, circulate in the room made available to them, work in groups, use the blackboard, etc. taking care not to disturb the other classes.

## Role of the teacher

- The teacher will give the statement sheets to the students.
- The teacher will inform students in classes competing in the Junior category (Nat. 5) that they do not have to consider exercises 11-13.
- The teacher can help students:
$\checkmark$ thoroughly read the statements and instructions given for each exercise;
$\checkmark$ form groups;
$\checkmark$ choose methods and strategies;
$\checkmark$ confront opinions and criticize solutions before final drafting;
$\checkmark$ Maximize the participation of each student and remember that even partial solutions (if not a complete solution) will be examined.
- Following the Discovery test, the teacher will be able to help the class in order to best prepare for the official test.

No submission of the results of the discovery test is requested. However, you can send photos of remarkable solutions, your comments and suggestions to: alan.walker@uws.ac.uk

Reminder of the website address : https://www.uws.ac.uk/about-uws/a-21st-century-university/partnerships/math\�\�matiques-sans-fronti\�\�res/

## Discovery Test 2023



Your solution is to be written in French, German, Spanish, or Italian using a minimum of 30 words.

Amandine möchte mit dem Fahrrad fahren, das ihre Freundin ihr ausgeliehen hat. Leider hat sie den dreistelligen Zahlencode für das Fahrradschloss vergessen. Mit Geduld und Methode versucht sie, die richtige Zahlenkombination wiederzufinden, indem sie eine Kombination nach der anderen testet.

Jeder Test mit einer Kombination dauert ungefähr zwei Sekunden. Amandine denkt, dass sie kaum Chancen hat, die richtige Kombination in weniger als 30 Minuten zu finden.

Denkt ihr das auch?
Begründet eure Antwort.

Amandine quiere usar la bicicleta que le ha prestado su amigo. Por desgracia no se acuerda del código de tres cifras del candado antirrobo.
Con mucha paciencia, va ensayando paso a paso las posibilidades para recuperar la combinación.

Cada prueba le lleva aproximadamente dos segundos. Amandine piensa que tiene pocas posibilidades de encontrar la combinación correcta en menos de 30 minutos.
¿Está de acuerdo con ella?
Justifique la respuesta.


Amandine veut utiliser le vélo que sonami lui a prêté. Malheureusement, elle a oublié le code de trois chiffres de l'antivol. Avec patience, elle procède méthodiquement par essais successifs pour retrouver cette combinaison.

Chaque essai lui demande environ deux secondes.Amandine pense qu'elle a peu de chance de trouver la bonne combinaison en moins de trente minutes.

Êtes-vous d'accord avec elle?

Justifiez votre réponse.

Amandine desidera usare la bicicletta prestatale da un suo amico. Sfortunamente ha dimenticato il codice di tre cifre dell'antifurto. Procede pazientemente e metodicamente con tentativi successivi per recuperare la combinazione. Per ogni tentativo impiega circa 2 secondi.

Amandine ritiene di avere poca possibilità d'individuare in meno di trenta minuti la combinazionz corretta.

## Siete d'accordo con lei?

Motivate la vostra riposta.


Nine tokens are numbered on the tails side from 1 to 9 . On the tails side, they are indistinguishable. They have been turned over and arranged as shown in the figure opposite. On some overlaps is the sum of the numbers of the two tokens that are superimposed there.

Write the token's number on each token.


Alexandre really likes to build paper models. He built the model of a podium represented above in perspective from a square sheet squared with sixteen squares.

He slashed this sheet along eight sides of squares while making sure to keep it in one piece.

Reproduce the grid with the numbers correctly oriented on the answer sheet and indicate in red the cuts made for this realization. Repeat such a grid to make Alexander's model and give it to your maths teacher.


Sabine tries to trace over the beams of the gable of the drawing of her little house without lifting the pencil and without going over a previously drawn segment.

She succeeds quite quickly and, after counting the number of segments from each named point, she understands that there are only two possible points for the beginning or the end of the path.

## What are the starting points in each

 figure?Respecting Sabine's rules of the game, draw the line without lifting the pencil on the second house, indicating the stages of the line.



The figure opposite is a rectangle made up of thirteen squares. The lengths of the sides of the squares are unique whole numbers of millimetres. Some lengths are given in the figure.

Find the length of the sides of all the squares by explaining your reasoning.

Reproduce the figure in real size.


Find all pairs of positive integers a and b, a greater than $b$, such that by adding their sum, their product and their difference, we find 2023.

## Explain your reasoning

Mélodie wants to do a puzzle. The lid of the box of this puzzle presents its rectangular pattern with the inscription: "1000 pieces". The grid of the puzzle, consisting of curved lines roughly following two perpendicular dimensions, can be considered as a grid. Melody first sets aside all the edge pieces. She finds exactly 124 edge pieces, including the four corners. While trying to put them together, Mélodie suddenly realizes that it is impossible for this puzzle to have exactly 1,000 pieces.

What is the minimum number of pieces in Melody's puzzle, knowing that the number of pieces is greater than 1,000 ?

## What are its dimensions?

Explain your reasoning.


The faces of a tetrahedron are two equilateral triangles of side 5 cm and two right triangles.

Build a pattern of this tetrahedron on the answer sheet.


Lucie's favourite game consists of nine pieces. Each piece has its place in the storage box. Unfortunately, there are only three pieces left, shown opposite.
Lucie wants to make new parts to replace the missing parts.

By assembling three pieces of the same row or of the same column or of the same diagonal, one can each time form a square. All the squares obtained have the same dimension.

We have the right to turn and re-turn the pieces.

Find the shape of the six lost pieces and place them in the box.


The figure above shows a cube with edge $a$, a small cube with edge $b$ and six cobblestones whose edges measure a or b.

Express the volume of each piece in terms of $\mathbf{a}$ and $\mathbf{b}$.
Position the seven pieces around the cube with edge $b$ so that the assembly of the eight pieces forms a large cube.

Represent this assembly in cavalier projection.
Write the expression for the volume of the large cube in two different ways in terms of $\mathbf{a}$ and $\mathbf{b}$ to obtain a remarkable equality.

## SENIOR LEVEL



The centurion orders his legionnaires: "Arrange in 4s! ". The legionnaires comply, but the last row is incomplete: it has only three legionnaires.
"Go into 5s!" then yells the centurion; but in the last rank is incomplete, there are again three legionnaires.

"Well, line up by 7s! ". Once again, the last row remains incomplete: there are still three legionnaires there.

How many legionnaires are there in the centurion's cohort, knowing that they are less than two hundred? Explain your reasoning.

Propose to the centurion an arrangement of his men in a rectangle where each will find his place.


A square is cut along two perpendicular straight lines into four identical pieces. These four pieces can be arranged to make a larger square, creating an empty square in the middle.

From a square with a side of 8 cm , make such a cutout so that the five regions of the large square have
 the same area.

Glue the four cut-out pieces to the answer sheet, forming the large square in the figure on the right.
Calculate the exact values of the lengths of the four sides of the cut pieces.


We build a sequence of numbers. 2022 is the first. The second is obtained by summing the squares of the digits of 2022, namely: $22+02+22+22$ which gives 12 . We continue like this. The third number is then 5 and the fourth 25 .

Calculate the first twenty numbers.
What will be the $2,023^{\text {rd }}$ number?
Explain your reasoning.


## Exercise 1-Try your luck - 7 points -

From 000 to 999 , there are 1,000 possible codes for this padlock.
At a rate of 2 s per test, Amandine can test 900 codes.
The probability of finding the right code in less than half an hour is then $90 \%$.
Amandine thinks she has little luck when she has every chance!

## Objectives and competencies:

## Seek to reason Communicate

Enumeration, probabilities, duration conversions, time, interpretation

## Proposed scale:

3 pts for language
4 pts for reasoning and conclusion
Any form of coherent research will be valued by at least 2 pts.

## Exercise 2-Head Tokens - 5 points -

## Objectives and competencies:

Search Calculate
Calculation, operations, logic, trial and error.

| Proposed scale: |
| :--- |
| 1 pt for a start of research |
| 2 pts for half of the correct values |
| 2 pts for the rest |




## Exercise 3 - The podium cup - 7 points -



The shaded cell will be used to consolidate the model during collage...
The numbers are only used to understand the corrected for cutting.
Objectives and competencies:
Seek to reason Represent Model
Puzzle, space, pattern, puzzle, cutting, model

| Proposed scale: |
| :--- |
| 3 pts for well-placed and oriented numbers |
| 2 pts for red cutting lines |
| 2 pts for the model |
| There are several solutions. |

## Exercise 4 - Beam in the eve - 5 points -

Let us call "degree" of a point the number of segments to be drawn of which this point is an end. The only possible points for the beginning or end of a path are the odd "degree" points. In the first figure the only points that allow you to start are C and D.

In the second figure, only the points F and I are of odd "degree", and therefore will make the trace respecting Sabine's rule of the game.
For example, we can trace the FBAEGDHGIHCBIEFI circuit.


Objectives and competencies:
Search Raisonner Represent
Graph, parity, diagram, circuit, plot, trial and error.

## Proposed scale:

2 times 2 pts to find the two starting points of each house 1 pt for a plot of the second house

## Exercise 5 - Almost a thousand - 7 points -

If we note the number of pieces on the length and the number $x$ on the width, the total number of pieces of the edge is: because it is not necessary to count twice the $y$ pieces of the $x+y+x+y-4=124$ corners. Hence $x+y=64$.
The minimum number of pieces of Melody's puzzle is 1,008 .
There are 36 pieces on the length and 28 pieces on the width.

| $x$ | and | $x y$ |
| :---: | :---: | :---: |
| 35 | 29 | 1015 |
| 36 | 28 | 1008 |
| 37 | 27 | 999 |
| 38 | 26 | 988 |

## Objectives and competencies:

Search Reason Calculate
Arithmetic, multiples, equation, multiplication, literal calculus, logic

## Proposed scale:

2 pts for an equation
2 pts to think about "minus 4"
2 pts for reasoning
1 pt for solution
Any form of coherent research will be valued by at least 2 pts.

## Exercise 6 - All squares - 5 points -



Objectives and competencies:
Search Reason Calculate
Length calculation, logic, hole operations

The difficulty of this exercise is to find the first square whose side we want to look for.
Here is a possible approach to find the solution.
The square (1) has $22 \mathrm{~mm}(84-62)$
The square (2) has for side 40 mm (62 22)

The (3) square has $78 \mathrm{~mm}(62+40+48-$ 72)

The square (4) has $6 \mathrm{~mm}(78-72)$
The square (5) has for side 18 mm (4022)

The square © has for side $10 \mathrm{~mm}(40+$ 18-48)
The square $\mathbb{7}$ has for side $28 \mathrm{~mm}(18+$ 10)

The square (8) has for side 38 mm (10 + 28)

The square (9) has for side $66 \mathrm{~mm}(28+$ 38)

## Proposed scale:

2 pts for the length of the first square
2 pts for other lengths
1 pt for full-scale figure
Any form of coherent research will be valued by at least 2 pts.

## Exercise 7 - With letters - 7 points -

$$
\begin{gathered}
(a+b)+a \times b+(a-b)=2023 \\
2 a+a \times b=2023 \\
a(2+b)=2023
\end{gathered}
$$

The only divisors of 2,023 are $1 ; 7 ; 17 ; 119 ; 289$ and 2,023 .
Thus, $119 \times 17=2023$ or $289 \times 7=2023$ or $1 \times 2023=2023$
With $a>b>0$, there are two solutions : and $a=289$ therefore $=2+b=7 \boldsymbol{a} 289$ and $=\mathbf{5 b}$.

$$
a=119 \quad \text { and therefore }=2+b=17 \boldsymbol{a} \mathbf{1 1 9} \text { and } \boldsymbol{b}=\mathbf{1 5} .
$$

## Objectives and competencies:

## Reasoning Calculate

Algebra, equation, decomposition of a number, divisors, literal calculus.

| Proposed scale: |
| :--- |
| 1 pt for writing the equation |
| 3 pts for 2,023 divisors |
| 3 pts for the solution |

## Exercise 8 - Four Sides - 5 Points -

Here is a correct pattern among others:

## Objectives and competencies:

## Search reason represent

Pattern, solid, space, tetrahedron, triangle, path, compass, plane geometry, drawn geometry.

## Proposed scale:

1 pt for two correct faces
4 pts for a correct pattern

## Exercise 9 - Magic puzzle - 7 points -

Proceeding rigorously, several remarks:
The second column allows only one shape for box 8 .
Similarly, the descending diagonal allows only one shape for box 9 .
From there, deductions to complete the other boxes.


| Objectives and competencies: | (1) | (2) | (3) |  |
| :---: | :---: | :---: | :---: | :---: |
| Search Reason Represent Vision in the plan, assemblies, puzzle, | (4) | (5) | (6) |  |
| Proposed scale: <br> 1 pt per lost coin 1 pt for plot accuracy | (7) | © | (9) |  |
|  |  |  |  |  |

## Exercise 10 - Power 3 - 10 points - 3rd

Here is a cavalier perspective view of the assembly. You can see seven of the eight pieces that make it up: The edge cube $a$ is hidden.

The assembly found illustrates equality:

$$
(a+b)^{3}=a^{3}+3 a^{2} b+3 a b^{2}+b^{3}
$$



## Objectives and competencies:

## Search to reason to model

Geometry in space, literal calculation, remarkable identity, 3D puzzle

## Proposed scale:

3 pts for the expressions of the volumes of each piece
( 0.5 for cubes, 1 pt for $\mathrm{ab}^{2}, 1 \mathrm{pt}$ for $\mathrm{a}^{2} \mathrm{~b}$ )
3 pts for a cavalier perspective
2 pts for each expression of the volume of the large cube

## Exercise 11 - In rows, by Jupiter! - 5 points $2^{\text {nde }}$ -

Let $n$ be the number of legionnaires.
At each centurion attempt, there are always three legionnaires who are not put away.
It is deduced that $n-3$ is a multiple of 4 and 5 and 7
The smallest common multiple of $4 ; 5$ and 7 is 140 . Hence $n=143$.
There are 143 legionnaires in the centurion cohort.
$143=11 \times 13$
The centurion will have two possibilities to store his legionaries:
11 ranks of 13 legionnaires or 13 ranks of 11 legionaries.

## Objectives and competencies:

## Search Reason Communicate Calculate

Arithmetic, multiples, multiplication tables, logic

## Proposed scale:

1 pt for answer: 143
3 pts for an explanation
1 pt for rectangle $11 \times 13$
Any form of coherent research will be valued by at least 2 pts.

## Exercice 12-Opening - 7 points -



The area of the first square is $64 \mathrm{~cm}^{2}$.
It is cut into four equal parts.
The area of the central square of the second figure is then $16 \mathrm{~cm}^{2}$, its side measures 4 cm.

We obtain the system:

$$
\left\{\begin{array}{l}
x+y=8 \\
x-y=4
\end{array}\right.
$$

The solution is $x=6$ and $y=2$.

The four parts have their other two sides of the same length and are valid in this case:

$$
\frac{\sqrt{80}}{2}=2 \sqrt{5}
$$

## Objectives and competencies:

## Search Reason Calculate Model

Literal calculus, equations, area, system, puzzle, cutting

## Proposed scale:

2 pts for an equation
2 pts for the torque solution
2 pts for lengths on the other two sides
1 pt gluing of the assembly
Any form of coherent research will be valued by at least 3 pts.

## Exercise 13-This year! - 10 points

We must start calculating the first terms and observe:

| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Ǹ |  | $\begin{gathered} \text { n } \\ \text { II } \\ \text { I } \\ \text { I } \end{gathered}$ | $\begin{aligned} & \text { n } \\ & \text { II } \\ & \text { n } \end{aligned}$ | $\begin{gathered} \text { N} \\ \text { II } \\ \text { in } \\ \text { ì } \end{gathered}$ | $\begin{gathered} n \\ \infty \\ 11 \\ \vdots \\ \vdots \\ \text { a } \\ \text { N } \end{gathered}$ | $\begin{gathered} \infty \\ \text { i } \\ 1 \\ i \\ + \\ \infty \end{gathered}$ |  | $\begin{gathered} \text { T} \\ \text { II } \\ \text { in } \\ \text { + } \\ + \\ + \\ \hline \end{gathered}$ | $\begin{gathered} \text { N} \\ \text { II } \\ \text { N } \\ \text { I } \end{gathered}$ | $\begin{aligned} & 广 \\ & \text { II } \\ & \text { I } \\ & \text { N } \\ & \text { N } \end{aligned}$ | $\begin{aligned} & 0 \\ & \text { II } \\ & \text { I } \end{aligned}$ |  | $\begin{gathered} \infty \\ \stackrel{\infty}{n} \\ \stackrel{1}{1} \\ \underset{\sim}{+} \\ \text { n } \end{gathered}$ | 2 $\infty$ 11 c + + 1 |

We see that the 15th number is the same as the 7th. As the rule remains the same, we know that the 23rd, 31st, 39th numbers are also 89 . From rank 7 , the sequence is period 8 , so the 8 th, 2,000th , 2,024th numbers are 145. And the 2023rd number is 89 .

## Objectives and competencies:

## Seek to reason communicate calculate Represent

Square of a number, sums of squares, perfect squares, sequence of numbers, period, arithmetic.

## Proposed scale:

3 pts for the calculation of the first 20 values.
3 pts for the period of 8
4 pts for the calculation of the $2,023^{e}$ and reasoning
Any form of coherent research will be valued by at least 4 pts.

This document contains 17 pages.

